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Knut Einar Rosendahl and Halvor Briseid Storrøsten

Emissions trading with updated grandfathering

Entry/exit considerations and distributional effects

Abstract:
Allocation of free emissions allowances may distort firms' incentives or have adverse distributional effects. Nevertheless, Böhringer and Lange (2005) show that in a closed emissions trading scheme with a fixed number of firms, a first-best outcome can be achieved if the base year for allocation is continually updated (i.e. updated grandfathering). In this paper we examine whether updated grandfathering alters the entry and exit conditions for firms compared to pure grandfathering, and how the distributional effects are affected. We find that updated grandfathering functions surprisingly similar to pure grandfathering: First, the incentives to entry and exit are identical under the two regimes. Second, the total value of free quotas to existing firms, based on emissions before the system starts, is identical under pure and updated grandfathering. In both cases, higher prices under updated grandfathering exactly match the shorter time period with free allowances. The only difference occurs when there is some combination of auction and pure or updated grandfathering, in which case the total value of free quotas will always be highest under pure grandfathering. Entry and exit incentives are still the same.

Keywords: Emission trading, Allocation of quotas, Quota prices

JEL classification: H21, Q28

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Address: Knut Einar Rosendahl, Statistics Norway, Research Department.
E-mail: knut.einar. rosendahl@ssb.no

Halvor Briseid Storrøsten, Statistics Norway, Research Department.
E-mail: halvor.storrosen@ssb.no

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1 Introduction

In response to the growing concern for climate change due to emissions of greenhouse gases (see, e.g., IPCC, 2007), there has been increased interest in emissions trading systems and different schemes for allocation of emissions allowances. Besides capping emissions at a certain level, these schemes may affect both distribution of profits and market efficiency. It is well known that a competitive emissions trading market will be cost-effective if quotas are auctioned or lump sum distributed (Montgomery, 1972). In practice, however, concern for competitive industries makes auctioning of quotas politically difficult. Furthermore, allocating free quotas in a lump sum manner (e.g., based on emissions before the system was initiated, so-called 'pure grandfathering'), may lead to unwanted distributional effects in the long run. For instance, it may seem unreasonable to transfer free quotas to firms that have closed down long ago, whereas new firms have to pay their full quota costs.

As an example, the EU Emissions Trading Scheme (EU ETS) has a mixture of allocation rules. In the second period, i.e., 2008-12, allocation of free quotas to existing firms are mostly based on historic emissions levels before the system was initiated. Many Member States also allocate quotas based on emission levels in the first year of period 1 (2005) (Neuhoff et al., 2006). Moreover, special rules for firm closure and new entrants are incorporated (EU, 2003, 2005; Watanabe and Robinson, 2005).

In this perspective, it is interesting that Böhringer and Lange (2005) show that a truly lump-sum allocation is not necessary to achieve cost-effectiveness in a closed emissions trading system. By updating the baseyear(s) for allocation continually (referred to as 'updated grandfathering'), so that allocation of allowances is proportional to emissions e.g. k years ago, cost-effectiveness is achieved even though the firms observe that their current emissions affect their future allocation of quotas. What happens is simply that the current price is bid up until it equals marginal abatement cost plus expected benefits from future allowances. As long as the allocation rate is identical across all firms, and firms share expectations about future prices, marginal abatement costs among firms become equal. Rosendahl (2008) points to several factors that may alter Böhringer and Lange's result. For instance, the quota price may become infinitely large if the marginal abatement costs grow too fast. He also shows that with banking and borrowing allowed, updated grandfathering does not in general lead to a dynamically cost-effective solution, as opposed to pure grandfathering or auctioning.

In the analysis by Böhringer and Lange (2005) and Rosendahl (2008) there is no entry or exit of firms. Thus, an essential question is whether updated grandfathering leads to other incentives with respect to entry and exit compared to pure grandfathering (or auction). After all, one of the prime motives for avoiding pure grandfathering is the different treatment of existing and new firms. Updated grandfathering gives new firms free allocation of quotas after a few years, whereas old firms no longer receive allowances into the future if they close down. Thus, one might expect that the entry and exit conditions are different with updated grandfathering.

This paper adds to previous analysis of updated grandfathering by explicitly capturing the incentives for entry and exit. We limit the analysis to a closed system for emissions trading with binding emissions target and perfect competition in both the product and the emissions markets. It should be noted that an open system with binding caps on the amount of imported quotas will mimic the mechanisms of a closed system, as abatement costs and quota prices will be similar to what we would have in a closed system where the emissions cap is increased by the allowable amount of imported quotas in the open system. This simple observation increases the relevance of our analysis.[1]

By extending the analytical framework set up by Böhringer and Lange (2005), we first show that the incentives for entry and exit are in fact similar under pure and updated grandfathering. That is, updated grandfathering does not give new firms more incentives to invest compared to pure grandfathering (or auction), even though they receive free allowances within few years if they invest. This result also holds with a combination of auctioning and grandfathering.

Second, we compare the expected lump sum transfers in form of free quotas under pure and updated grandfathering. We show that the *total* value of free quotas to old firms are identical under pure and updated grandfathering if there is no auctioning of quotas. Moreover, it is possible to construct an allocation rule under pure grandfathering that makes *each* firm's value of free quotas identical under the two regimes. Thus, updated grandfathering gives the same distributional effects to old and new firms as pure grandfathering. On the other hand, if there is some combination of auction and pure or updated grandfathering, the total value of free quotas will always be higher under pure grandfathering than under updated grandfathering (given that the auction rate is the same under the two regimes).

Besides Böhringer and Lange (2005) and Rosendahl (2008), few pa-

[1]For instance, in the EU ETS there is an upper limit to the use of CDM or JI credits (i.e., project credits under the Kyoto Protocol). Thus, as long as the price of such credits is below the quota price within the EU ETS, the imports of such credits will be given, either by the cap on such imports or the access to CDM and JI credits.

pers have analyzed the consequences of updated grandfathering. Keats Martinez and Neuhoff (2005) use a two-period model to demonstrate how updating can distort the allowance price, leading to inefficiencies if different firms face different allocation rates or have different discount rates. Åhman et al. (2007) propose a ten-year updating rule, claiming that a ten-year lag would significantly weaken this distortion because of discounting of the value of future allowances. Numerical simulations of different allocation rules, including updated grandfathering, are presented in Burtraw et al. (2006), applied to a regional emissions trading market in the U.S. Mackenzie et al. (2008) generalize the analysis of Böhringer and Lange (2005) to initial allocation mechanisms based on inter-firm relative performance comparisons. They also show that it is possible to achieve social optimality by allocating permits based only on an external factor that is independent of production and emissions.

In the next section we set up the analytical model, and then we derive and discuss some theoretical results. Section 3 presents some simple numerical illustrations. In the last section we draw some conclusions that may be relevant for policy makers trying to find new and better ways to allocate quotas.

2 Theoretical analysis

Consider a closed emissions trading system that is implemented at time $t = 0$ and lasts for an unknown number of periods, with probability of ending equal to $(1 - b)$ in each period ($b \in [0, 1]$), and notification of ending is given at the end of the last period.[2] We assume risk neutral firms and that there is perfect competition in both the emissions market and the product markets that the firms operate in. We also assume free exit and entry. Let firm i's technology in period t ($t = 0, 1, ...$) be summarized by the cost function $c^{i,t} = c^{i,t}(q^{i,t}, e^{i,t}) + F^{i,t}$. Here $q^{i,t}$ and $e^{i,t}$ denote the production and emissions levels of firm i, whereas $F^{i,t}$ is a fixed annual cost. The cost function is twice differentiable and convex with $c_q^{i,t} = \partial c^{i,t}/\partial q^{i,t} > 0$; $c_e^{i,t} \leq 0$; $c_{qq}^{i,t}, c_{ee}^{i,t}, -c_{qe}^{i,t} \geq 0$; and $c_{qq}^{i,t} \cdot c_{ee}^{i,t} - \left(c_{qe}^{i,t}\right)^2 > 0$. A new firm must pay the sunk investment cost I^i before production can start. We assume that exit is equivalent to not paying the fixed annual cost $F^{i,t}$. Firm i sells output at price $p^{i,t}$, and all firms operate with the same discount factor δ. We assume that all firms have equal expectations about future quota prices (we come back to this assumption in the conclusions).

[2]It can be shown that the results of this analysis still applies if the system is known to end with certainty in a future period (with or without uncertainty in the intermediate time periods).

The government's environmental objective is to keep aggregate emissions below a threshold level such that $E^t \geq \sum_i e^{i,t}$. The emissions target is assumed binding in all periods, and no banking or borrowing is allowed. The quota price is denoted by σ^t under pure grandfathering and ς^t under updated grandfathering. We will label the firms that exist before $t = 0$ (and thus receive free quotas under pure grandfathering) as "old", whereas "new" refer to firms that consider to enter the market at some time t.

It is well known that pure grandfathering (as well as auctioning) leads to a cost-effective outcome of an emissions trading system, because the free quotas act as a lump sum transfer to the firms (see e.g. Montgomery 1972). As shown by Böhringer and Lange, updated grandfathering leads to the same cost-effective abatement if the system is closed and the number of firms is fixed. In the following we want to examine whether this result carries over in a model with endogenous entry and exit, i.e., does updated grandfathering give the same incentives with respect to entry and exit as does pure grandfathering (or auctioning)?

We will consider three optimization problems or conditions for each of the two systems. First, we will consider the optimization problem for an existing firm, given that it finds it profitable to stay in the market. This is just a replication of what Böhringer and Lange do, but it is useful to include for the rest of the analysis. The first order conditions that are derived are also valid for a new firm that has decided to enter the market. Second, we will look at the exit condition for an existing firm. Finally, we will examine the entry condition for a firm that considers to invest.

2.1 Market equilibrium under pure grandfathering

Under pure grandfathering firms receive free quotas based on their emissions in one or more base years before $t = 0$. Let $\overline{e}^i$ denote some weighted sum of firm i's emissions in the base year(s) for this regime, and let $\overline{E}$ denote aggregate emissions defined in the following way: $\overline{E} = \sum_i \overline{e}^i$. Furthermore, let $\gamma^{P,t}\overline{e}^i$ be the number of quotas firm i receives at time t (we will refer to $\gamma^{P,t}$ as the allocation rate). Lastly, let a, with $a \in [0, 1]$, refer to the fraction of auctioned quotas such that $a = 1$ ($a = 0$) refers to full (no) auctioning. Note that we must have $\gamma^{P,t} = (1 - a)\frac{E^t}{\overline{E}}$.

The maximization problem of firm i in some period s is the following:

$$\max \sum_{t=s}^{\infty} (\delta)^{t-s} \left[\pi^{i,t} - (b)^{t-s} \sigma^t (e^{i,t} - \gamma^{P,t}\overline{e}^i)\right],$$

where $\pi^{i,t} = p^{i,t}q^{i,t} - c^{i,t}(q^{i,t}, e^{i,t}) - F^{i,t}$. Obviously, for firms entering the market after the trading scheme has started, $\overline{e}^i = 0$.

When period t arrives, there is no longer any uncertainty whether the trading scheme ends before this period. Thus, we get the following optimality conditions for a firm that doesn't exit before period t:

$$p^{i,t} \leq c_q^{i,t} \text{ and } (p^{i,t} - c_q^{i,t})q^{i,t} = 0, \tag{1}$$

$$\sigma^t \leq -c_e^{i,t} \text{ and } (\sigma^t + c_e^{i,t})(e^{i,t} - e_{BaU}^{i,t}) = 0. \tag{2}$$

As is well known, cost efficiency requires that marginal costs are equal across firms with positive production and positive abatement, and this is fulfilled by equations (1) and (2).

The firm will exit the market at time $t = s$ unless the present value of expected profits is non-negative, i.e.:[3]

$$\sum_{t=s}^{\infty} (\delta)^{t-s} \left[\pi^{i,t} - (b)^{t-s} \sigma^t (e^{i,t} - \gamma^{P,t}\overline{e}^i)\right] \geq \sum_{t=s}^{\infty} (d)^{t-s} \sigma^t \gamma^{P,t}\overline{e}^i, \tag{3}$$

where $d = \delta b$. Note that, because the free quotas act as a lump sum transfer, their expected value appears at both sides of the equation. Consequently, they will not affect the exit condition of a rational profit maximizing firm.

Finally, the entry condition for a new firm at time $t = s$ is the following:

$$\sum_{t=s}^{\infty} (\delta)^{t-s} \left[\pi^{i,t} - (b)^{t-s} \sigma^t e^{i,t}\right] \geq I^i. \tag{4}$$

New firms will enter the market as long as the expected net present value of production exceeds the investment cost. Note that the entry and exit conditions lead to a cost effective outcome, similar to the outcome under auctioning.

Before turning to updated grandfathering, we notice that the expected net present value of free quotas to an old firm i received from time $t = s$ can be expressed as $(1-a)\frac{\overline{e}^i}{\overline{E}} \sum_{t=s}^{\infty} (d)^{t-s} \left(-c_e^t\right) E^t$, where $\left(-c_e^t\right)$ is the common marginal abatement costs realized in the market (cf. 2).

[3] Note that the firm will not exit if the inequality fail to hold for one or more periods, as long as the whole sum is non-negative. The firm may, however, have no production for some periods as long as it pays the fixed cost $F^{i,t}$.

2.2 Market equilibrium under updated grandfathering

Under updated grandfathering firms receive emissions in period t based on their emissions level in period $t-k$ (i.e., the base year is continually updated). Let $\gamma^t e^{i,t-k}$ be the number of quotas firm i receives at time t, so that $\gamma^t = (1-a)\frac{E^t}{E^{t-k}}$. With perfect competition, E^t is considered exogenously given for each firm.

Firm i now solves the following maximization problem:

$$\max \sum_{t=s}^{\infty} (\delta)^{t-s} \left[\pi^{i,t} - (b)^{t-s} \varsigma^t \left(e^{i,t} - \gamma^t e^{i,t-k}\right)\right].$$

The first order conditions are given by:

$$p^{i,t} \leq c_q^{i,t} \text{ and } (p^{i,t} - c_q^{i,t})q^{i,t} = 0, \tag{5}$$

$$\varsigma^t - (d)^k \varsigma^{t+k}\gamma^{t+k} \leq -c_e^{i,t} \text{ and } (\varsigma^t - (d)^k \varsigma^{t+k}\gamma^{t+k} + c_e^{i,t})(e^{i,t} - e_{BaU}^{i,t}) = 0. \tag{6}$$

As shown by Böhringer and Lange (2005), the outcomes in the interior solution are identical with respect to production and emissions under pure and updated grandfathering. The explanation is simply that both terms on the left hand side of inequality (6) are equal for all firms, so that marginal abatement costs must be equal for firms with positive abatement.[4] With total emissions fixed by the government, the same outcome is realized. Thus, the cost effective allocation is feasible under updated grandfathering.

The firm will exit the market at time $t = s$ unless the following condition holds:

$$\sum_{t=s}^{\infty} (\delta)^{t-s} \left[\pi^{i,t} - (b)^{t-s} \varsigma^t \left(e^{i,t} - \gamma^t e^{i,t-k}\right)\right] \geq \sum_{t=s}^{k+s-1} (d)^{t-s} \varsigma^t \gamma^t e^{i,t-k}. \tag{7}$$

The right hand side is the value of free quotas received over the next k years based on previous emissions. Except for net quota costs, this condition is identical to the condition (3) under pure grandfathering.

The entry condition at time $t = s$ under updated grandfathering is:

[4]The model of Böhringer and Lange (2005) does not include a discount factor or uncertainty about the future. If they differs across firms, or if expectations about quota prices differ, the result no longer holds. We return to this in the conclusions.

$$\sum_{t=s}^{\infty} (\delta)^{t-s} \left[\pi^{i,t} - (b)^{t-s} \varsigma^t \left(e^{i,t} - \gamma^t e^{i,t-k} \right) \right] \geq I^i, \tag{8}$$

where $e^{i,t-k} = 0$ for $t < k$.

Again, we see that the condition is the same as under pure grandfathering (cf. 4), except for the net quota costs. Recursive substitution of quota price in equation (6), remembering that marginal abatement costs will be equal among all abating firms (i.e., $-c_e^{i,t} = -c_e^t$), gives that ς^t can be expressed as:

$$\varsigma^t = \sum_{v=0}^{\infty} \left[(d)^{vk} \left(-c_e^{vk+t} \right) \prod_{w=0}^{v-1} \gamma^{(1+w)k+t} \right]. \tag{9}$$

We see that the quota price depends on the allocation rate and firms' marginal abatement costs in future periods. As shown by Rosendahl (2008), this price could be significantly higher than the price under pure grandfathering.

2.3 Comparing entry and exit in the two regimes

We now compare the net quota costs in the entry and exit conditions under the two regimes. As explained above, if we can show that the net quota costs are equal under the two regimes, we have shown that the entry and exit conditions are the same.

Starting with the entry decision, assume that firm i considers to enter the market in period $t = s$. Under pure grandfathering the expected present value of quota costs (QC_{Entry}^P) is equal to (cf. 4):

$$QC_{Entry}^P = \sum_{t=s}^{\infty} (d)^{t-s} \left(-c_e^t \right) e^{i,t}. \tag{10}$$

Note that QC_{Entry}^P also corresponds to firms' quota costs under auctioning. How does this compare with quota costs under updated grandfathering? The net present value of quota costs (QC_{Entry}^U) can be expressed as (cf. 8):

$$QC_{Entry}^U = \sum_{t=s}^{k+s-1} \left[(d)^{t-s} \varsigma^t e^{i,t} \right] + \sum_{t=k+s}^{\infty} \left[(d)^{t-s} \varsigma^t \left(e^{i,t} - \gamma^t e^{i,t-k} \right) \right]. \tag{11}$$

The left part of the expression refers to the periods before the new firm gets free quotas due to previous emissions. The right part refers to net quota costs when some quotas are received for free. We state the following lemma:

Lemma 1 *Assume perfect competition in both the quota and product markets. Then, if a firm decides to enter the market, the expected net present value of quota costs are the same under pure and updated grandfathering.*

Proof. Equation (11) can be reformulated as:

$$
\begin{aligned}
QC^{U}_{Entry} = & \sum_{t=s}^{\infty} \left[(d)^{t-s} \varsigma^{t} e^{i,t} \right] - \sum_{t=k+s}^{\infty} \left[(d)^{t-s} \varsigma^{t} \gamma^{t} e^{i,t-k} \right] \\
= & \sum_{t=s}^{\infty} \left[(d)^{t-s} e^{i,t} \sum_{v=0}^{\infty} \left[(d)^{vk} (-c_e^{vk+t}) \prod_{w=0}^{v-1} \gamma^{(1+w)k+t} \right] \right] - \\
& \sum_{t=k+s}^{\infty} \left[(d)^{t-s} \gamma^{t} e^{i,t-k} \sum_{v=0}^{\infty} \left[(d)^{vk} (-c_e^{vk+t}) \prod_{w=0}^{v-1} \gamma^{(1+w)k+t} \right] \right] \quad (12)
\end{aligned}
$$

where we have inserted the quota price from equation (9). The first part of the equation refers to the expected (gross) quota costs, whereas the second part refers to the expected present value of free quotas under updated grandfathering. Splitting the inner sum in the first part of the equation we get:

$$
\begin{aligned}
QC^{U}_{Entry} = & \sum_{t=s}^{\infty} \left[(d)^{t-s} e^{i,t} (-c_e^{t}) \right] + \\
& \sum_{t=s}^{\infty} \left[(d)^{t-s} e^{i,t} \sum_{v=1}^{\infty} \left[(d)^{vk} (-c_e^{vk+t}) \prod_{w=0}^{v-1} \gamma^{(1+w)k+t} \right] \right] - \\
& \sum_{t=k+s}^{\infty} \left[(d)^{t-s} \gamma^{t} e^{i,t-k} \sum_{v=0}^{\infty} \left[(d)^{vk} (-c_e^{vk+t}) \prod_{w=0}^{v-1} \gamma^{(1+w)k+t} \right] \right] \quad (13)
\end{aligned}
$$

The first part is equal to the quota costs under pure grandfathering (cf. 10). The second part of equation (13) refers to the discounted value of the increase in quota price times the firms' emissions in the future periods. That is, the firms' extra payment due to higher quota prices under updated grandfathering. It remains to show that this second part of the equation is exactly equal to minus the third part, i.e., the value of free quotas. The second part of (13) is equal to:

$$
\begin{aligned}
& \sum_{t=s}^{\infty} \left[(d)^{t-s+k} \gamma^{k+t} e^{i,t} \sum_{v=0}^{\infty} \left[(d)^{vk} (-c_e^{(1+v)k+t}) \prod_{w=1}^{v} \gamma^{(1+w)k+t} \right] \right] \\
= & \sum_{t=k+s}^{\infty} \left[(d)^{t-s} \gamma^{t} e^{i,t-k} \sum_{v=0}^{\infty} \left[(d)^{vk} (-c_e^{vk+t}) \prod_{w=0}^{v-1} \gamma^{(1+w)k+t} \right] \right].
\end{aligned}
$$

Hence, the two lower equations in (13) cancel out and we are left with the first part of the equation, which is equal to equation (10). This proves the lemma. ■

Next, we consider the net quota costs for an existing firm that considers to exit the market. What matters here is the *difference* in quota costs with and without exit. We state the following lemma:

Lemma 2 *Assume perfect competition in both the quota and the product markets. Then, if a firm decides to exit the market, the effect on expected net present value of quota costs is the same under pure and updated grandfathering.*

Proof. Under pure grandfathering the difference in net quota costs with and without exit (QC^P_{Exit}) is exactly equal to $-QC^P_{Entry}$, i.e., the net quota costs of entering the market (cf. 2, 3 and 10). This is not surprising, as free quotas act as a lump sum transfer to old firms.

Under updated grandfathering, the difference in net quota costs with and without exit (QC^U_{Exit}) is given by (cf. 7):

$$QC^U_{Exit} = -\sum_{t=s}^{s+k-1} (d)^t \varsigma^t \gamma^t e^{i,t-k} - \sum_{t=s}^{\infty} \left[(d)^t \varsigma^t \left(e^{i,t} - \gamma^t e^{i,t-k} \right) \right]. \quad (14)$$

The first part is the gross quota costs of exit, which are negative and reflects that the firm receives free quotas the first k years if it exits. The second part is the quota costs if the firm stays in the market, which is subtracted from the gross costs of exit. By splitting the second sum we get:

$$\begin{aligned} QC^U_{Exit} = &- \sum_{t=s}^{s+k-1} (d)^t \varsigma^t \gamma^t e^{i,t-k} - \sum_{t=s}^{s+k-1} \left[(d)^t \varsigma^t \left(e^{i,t} - \gamma^t e^{i,t-k} \right) \right] \\ &- \sum_{t=s+k}^{\infty} \left[(d)^t \varsigma^t \left(e^{i,t} - \gamma^t e^{i,t-k} \right) \right] \\ = &- \sum_{t=s}^{s+k-1} (d)^t \varsigma^t e^{i,t} - \sum_{t=s+k}^{\infty} \left[(d)^t \varsigma^t \left(e^{i,t} - \gamma^t e^{i,t-k} \right) \right]. \end{aligned} \quad (15)$$

Comparing with (11) we see that $QC^U_{Exit} = -QC^U_{Entry}$, and so it follows from Lemma 1 and the fact that $QC^P_{Exit} = -QC^P_{Entry}$ (see above) that $QC^U_{Exit} = QC^P_{Exit}$. Thus, we have proved the lemma. ■

Based on the two lemmas we state the following proposition regarding incentives for entry and exit under an updated grandfathering regime:

Proposition 1 *Consider a closed system for emissions trading, with binding emissions target and perfect competition in both the product and the emissions markets. Then, the incentives regarding entry and exit are equal under pure and updated grandfathering.*

Proof. Since the entry and exit conditions between the regimes differ only with respect to net quota price, the proposition follows directly from Lemma 1 and Lemma 2. ■

Note that the derivations are made without any restrictions on a, i.e., the level of auctioning. Thus, the proposition holds for any combination of auction and (pure or updated) grandfathering.

Why does updated grandfathering give the same incentives with respect to entry and exit as pure grandfathering? After all, a new firm that enters the market will only have to pay for all its quotas the first k years with updated grandfathering, compared to all years with pure grandfathering. And similarly, a firm that exits will only receive free quotas the first k years with the former regime. The explanation is that under updated grandfathering the quota price becomes higher than under pure grandfathering. Thus, a new firm must pay a higher bill over the first k years under updated grandfathering, and a lower bill afterwards. As shown above, these two effects cancel each other out.

According to the proposition, there will be no incentives for the firms to deviate from the cost effective allocation under updated grandfathering, even when entry and exit are endogenous. In other words, updated grandfathering does not give new firms any better conditions than pure grandfathering or auction, and the effects on production and employment are the same. This might come as a surprise to policy makers trying to find new and better ways to allocate quotas.

2.4 Comparing the lump sum transfer in the two regimes

It is also interesting to look at the expected lump sum transfer to old firms under the two regimes, i.e., the value of free quotas received based on emissions before the trading system starts. Are these transfers also equal, as the entry and exit conditions are the same? Obviously, we must assume here that the auction rate a is the same across the two regimes.

The expected lump sum transfers ($LST^{i,j}$) with pure and updated grandfathering are given by the right hand sides of equations (3) and (7), respectively. As mentioned at the end of Section 2.1, under pure grandfathering $LST^{i,P}$ can be expressed as:

$$LST^{i,P} = (1-a)\frac{\overline{e}^i}{\overline{E}} \sum_{t=s}^{\infty} \left[(d)^{t-s} (-c_e^t) E^t \right]. \tag{16}$$

With updated grandfathering, we have (see appendix A):

$$\begin{aligned} LST^{i,U} &= (1-a) \sum_{t=s}^{k+s-1} \left[(d)^{t-s} \varsigma^t e^{i,t-k} \frac{E^t}{E^{t-k}} \right] \\ &= (1-a) \sum_{t=s}^{k+s-1} \left[(d)^{t-s} \frac{e^{i,t-k}}{E^{t-k}} M_t \right], \end{aligned} \tag{17}$$

where $M_t = \sum_{v=0}^{\infty} \left[(d)^{vk} (-c_e^{vk+t})(1-a)^v E^{vk+t} \right]$ is equal across firms.

How does the total lump sum transfer to all firms compare under the two regimes? From (17) we get (see appendix A):

$$\sum_i LST^{i,U} = \sum_i LST^{i,P} - N, \tag{18}$$

with

$$N = (1-a) \sum_{t=s}^{k+s-1} \left[(d)^{t-s} \sum_{v=0}^{\infty} \left[(d)^{vk} (-c_e^{vk+t}) \left[1 - (1-a)^v\right] E^{vk+t} \right] \right].$$

First, note that $N = 0$ if $a = 0$ or $a = 1$. This entails that the aggregate lump sum transfers are equal in the two regimes under zero auctioning. Second, we show in Appendix A that $0 < N < \sum_i LST^{i,P}$ if $a \in (0,1)$. Thus, with a combination of auction and (pure or updated) grandfathering, the total lump sum transfer is highest in the case with pure grandfathering.

Although we have shown that the *total* lump sum transfer to all firms is equal when $a = 0$, the lump sum transfer to an *individual firm* may not necessarily be equal across the two regimes. This depends crucially on how the allocation rule under pure grandfathering is designed. So far we have only assumed that the allocation of quotas is proportional to a weighted sum of historic emissions.[5] If we set equation (16) equal to equation (17), and solve for $\overline{e}^i$, we get:

[5]Note that the derivations above would hold for any allocation of quotas under pure grandfathering, given that the quotas are considered as lump sum transfers.

$$\overline{e}^i = \frac{\overline{E}}{\sum_{t=0}^{k-1}\left[(d)^t M_t\right]} \sum_{t=0}^{k-1}\left[(d)^t M_t \frac{e^{i,t-k}}{E^{t-k}}\right]. \qquad (19)$$

Thus, if the allocation rule under pure grandfathering is chosen in line with equation (19), the lump sum transfer to each firm will be identical to the lump sum transfer under updated grandfathering. Note that the right hand side of the equation is a weighted sum of the firms' emission levels in the k years before the system starts, where the weights are equal across firms.

Based on this we state the following proposition:

Proposition 2 *Consider a closed emission trading system with binding emissions target and perfect competition in both the product and the emissions market. Then, we have the following:*
i) The expected total value of free quotas to old firms based on emissions before the trading system starts are identical under pure and updated grandfathering if and only if there is no auctioning of quotas. Moreover, it is possible to construct an allocation rule (given by 19) under pure grandfathering that makes each firm's value of free quotas identical under the two regimes.
ii) If there is some combination of auction and pure or updated grandfathering, then the expected total value of free quotas to old firms based on emissions before the trading system started, will always be higher under pure grandfathering than under updated grandfathering (given that the auction rate is the same under the two regimes).

Proof. The proof follows from the derivations above. ■

The lump sum transfer is lower under updated grandfathering because the quota price falls when the auction rate increases. The quota price falls because the future value of current emissions decreases when the allocation rate is reduced (cf. 6). With pure grandfathering, the quota price is of course unaffected by the auction rate (cf. 2). Consequently, as the number of free quotas is reduced, the lump sum transfer is mostly reduced under updated grandfathering.

3 Numerical illustration

In this section we briefly examine two issues within a simple numerical model. First, although net present value of quota costs are identical for pure and updated grandfathering, the annual costs will differ. Thus, we illustrate how annual net quota costs may evolve over time under the two regimes. Second, we showed above that the lump sum transfer to old firms differ between the two regimes if and only if the auction rate

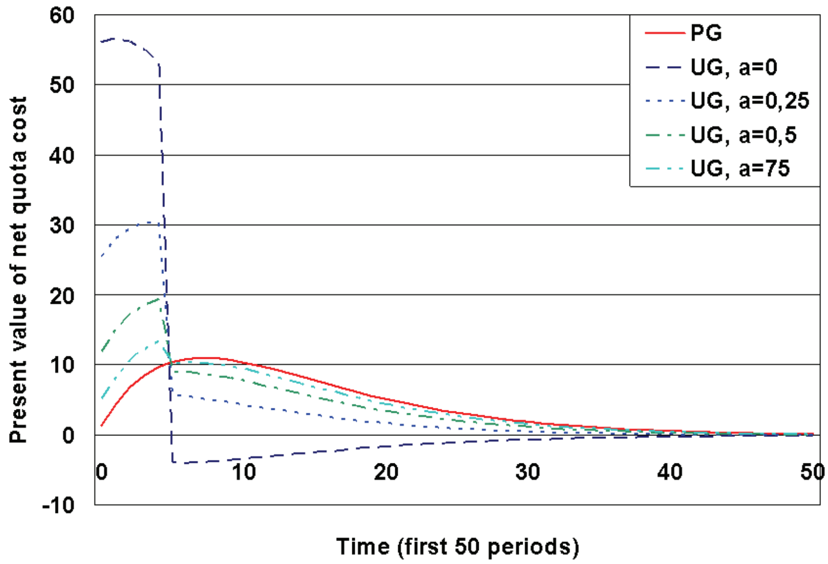

Figure 1: Time profile of discounted net quota cost with various rates of auctioning (new firms)

is strictly between 0 and 1. Here we illustrate how the transfer may depend on the auction rate under pure and updated grandfathering.

The numerical model is the same as in Rosendahl (2008), except that it is extended to allow for entry and exit of firms (see Appendix B for more details).

Consider first a firm that enters the market in year 0. Figure 1 shows the development over time of its discounted net annual quota costs under the two regimes, where PG and UG refer to pure and updated grandfathering respectively. Under updated grandfathering, the quota price and thus the annual quota costs depend on the auction rate, and so we illustrate the effects of various rates of auctioning in the figure.

In the pure grandfathering regime, the new firm never receives any free quotas and face the same conditions as under auctioning. In our simulation, the discounted value of annual quota costs first increases because the quota price rises quite strongly due to increased demand and a constant emissions target. Eventually, as the future quota costs are discounted, the present value starts to fall towards zero.

Just as under pure grandfathering, a new firm does not receive any free quotas the first k years under updated grandfathering (cf. 11). Still, we see that the annual quota costs differ markedly between the two regimes in these first years. As explained in Section 2.2, the quota

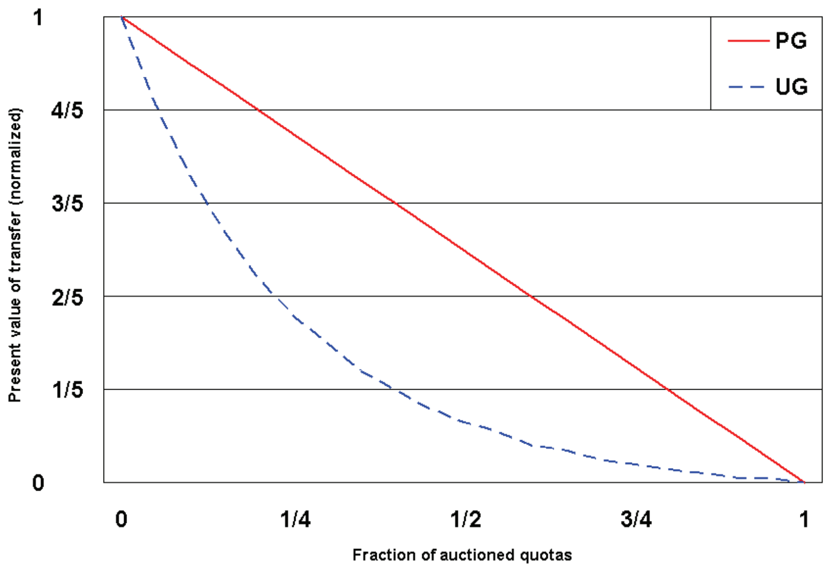

Figure 2: Lump sum transfer with various auction rates

price is higher under updated grandfathering and increases substantially as the fraction of auctioned quotas fall. The limiting cases are zero auctioning ($a = 0$) and full auctioning ($a = 1$), where the latter case has the same effects on net quota costs as pure grandfathering. After the first k years the net quota costs are substantially lower as the firm receives free allowances based on its earlier emissions. In our simulations we see that net quota costs become negative when the auction rate is zero. This is because the firm becomes a net quota seller as newer firms enter the market.

The figure shows quite clearly that new firms face much higher quota costs initially under updated grandfathering than under pure grandfathering (or auctioning). After the first k years, however, costs are much lower, so that total costs remain the same for all $a \in [0, 1]$ (note that the area under the curves correspond to the total present value of net quota costs).[6]

From Proposition 4 we know that the total value of free quotas ($LST^{i,j}$) will always be higher under pure grandfathering than under updated grandfathering, given some auction rate $a \in (0, 1)$. This is illustrated in Figure 2, where the discounted value of free quotas is normalized to one (at $a = 0$). We see that the value of the free quotas drop

[6]There is a long tail with a horisontal asymptote towards zero to the right. This is capped in Figure 1.

quite significantly under updated grandfathering when some auctioning is introduced (the exact curvature of the UG-curve depends of course on the different parameters of the model). The counterpart of this is that the government receives a higher income from a given (partial) auction rate under updated grandfathering compared to pure grandfathering.

4 Conclusions

It is well known that allocation of free emissions allowances may distort firms' incentives or have adverse distributional effects. Whereas pure grandfathering leads to a cost-effective outcome, it creates an asymmetry between old and new firms as only the former group of firms may receive free quotas. Thus, it is of interest when Böhringer and Lange (2005) show that free allowances based on updated grandfathering also may be cost-effective (within a closed system). With such a regime, new firms also receive free quotas after a fixed number of years, and so it may seem to be a more fair treatment of different firms. However, as Böhringer and Lange's analysis is undertaken without incorporating entry and exit of firms, a reasonable question is whether (dynamic) cost-efficiency still applies if entry and exit conditions are taken into account. Moreover, how are the distributional effects across firms under updated grandfathering?

In our paper we have shown that incentives regarding entry and exit are actually equal under pure and updated grandfathering (and auctioning). This is because the quota price is higher under updated grandfathering. New firms therefore have to pay a higher bill initially, but are better off later on when they earn the right to receive free quotas. The expected net present values of these two effects cancel out. The same reasoning applies to exit of existing firms. Thus, updated grandfathering may be dynamically cost-effective in a closed system.

Regarding distributional effects, we examine the size of the expected lump sum transfers coming from free allowances based on emissions before the trading system starts. We find that the total value of free quotas to old firms are identical under pure and updated grandfathering, given that there is no auctioning of quotas. Moreover, we show that it is possible to construct an allocation rule under pure grandfathering that makes each firm's value of free quotas identical under the two regimes. Again, the higher prices under updated grandfathering exactly compensate for the shorter period of time with lump sum allocation. On the other hand, if there is some combination of auction and pure or updated grandfathering, then the total value of free quotas will always be highest under pure grandfathering (given that the auction rate is the same under the two regimes). This means that public revenues from partial auction are highest when combined with updated grandfathering, which also follows from the fact that quota prices are highest under this regime.

Our analysis relies upon two important assumptions. First, we limit the analysis to a closed system with binding emissions target. As mentioned in the introduction, our results still apply if the system is open and there is a binding cap on the amount of imported quotas. If there is no such cap, Böhringer and Lange (2005) shows that updated grandfathering will not be cost-effective. Second, if discount rates or expectations about future quota prices differ across firms, their first order conditions will be affected differently by updated grandfathering. In this case the allocation is not cost-effective since the marginal abatement costs will differ among firms. This happens regardless of our assumptions about exit and entry.

The findings in this paper may be relevant for the ongoing policy debate regarding allocation schemes for emissions allowances. First, updated grandfathering does not give new firms, e.g., in the exposed industries, stronger incentives to invest than pure grandfathering or auctioning – their profits are the same in the long run. However, in the short run they are worse off with updated grandfathering. Second, the incentives to close down existing firms are unchanged when introducing updated instead of pure grandfathering. Third, if policy makers want to reduce the lump sum transfer to historic emitters following from pure grandfathering, initiation of an updated grandfathering scheme with no auctioning will not work as intended. Fourth, policy makers may ameliorate point three above by letting a fraction of the quotas be auctioned.

Our conclusion, therefore, is that updated grandfathering is at best identical to pure grandfathering with regard to efficiency and distribution effects. At worse, however, if our assumptions do not hold, updated grandfathering will no longer lead to a cost-effective outcome.

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A Appendix

Derivation of equation (17) (based on RHS of equation 7 and definition of γ):

$$
\begin{aligned}
LST^{i,U} &= (1-a)\sum_{t=s}^{k+s-1}\left[(d)^{t-s}\varsigma^{t}e^{i,t-k}\frac{E^{t}}{E^{t-k}}\right]\\
&= (1-a)\sum_{t=s}^{k+s-1}\left[(d)^{t-s}e^{i,t-k}\frac{E^{t}}{E^{t-k}}\left(\sum_{v=0}^{\infty}\left[(d)^{vk}(-c_e^{vk+t})\prod_{w=0}^{v-1}\gamma^{(1+w)k+t}\right]\right)\right]\\
&= (1-a)\sum_{t=s}^{k+s-1}\left[(d)^{t-s}e^{i,t-k}\frac{E^{t}}{E^{t-k}}\left(\sum_{v=0}^{\infty}\left[(d)^{vk}(-c_e^{vk+t})(1-a)^{v}\frac{E^{vk+t}}{E^{t}}\right]\right)\right]\\
&= (1-a)\sum_{t=s}^{k+s-1}\left[(d)^{t-s}\frac{e^{i,t-k}}{E^{t-k}}\left(\sum_{v=0}^{\infty}\left[(d)^{vk}(-c_e^{vk+t})(1-a)^{v}E^{vk+t}\right]\right)\right]\\
&= (1-a)\sum_{t=s}^{k+s-1}\left[(d)^{t-s}\frac{e^{i,t-k}}{E^{t-k}}M_t\right],
\end{aligned}
$$

where $M = \sum_{v=0}^{\infty}\left[(d)^{vk}(-c_e^{vk+t})(1-a)^{v}E^{vk+t}\right]$.
Derivation of equation (18) (based on equation 17):

$$
\begin{aligned}
\sum_i LST^{i,U} &= (1-a)\sum_i\left[\sum_{t=s}^{k+s-1}\left((d)^{t-s}\frac{e^{i,t-k}}{E^{t-k}}\sum_{v=0}^{\infty}\left[(d)^{vk}(-c_e^{vk+t})(1-a)^{v}E^{vk+t}\right]\right)\right]\\
&= (1-a)\sum_{t=s}^{k+s-1}\left[(d)^{t-s}\sum_{v=0}^{\infty}\left[(d)^{vk}(-c_e^{vk+t})\left[1-1+(1-a)^{v}\right]E^{vk+t}\right]\frac{1}{E^{t-k}}\sum_i e^{i,t-k}\right]\\
&= (1-a)\sum_{t=s}^{k+s-1}\left[(d)^{t-s}\sum_{v=0}^{\infty}\left((d)^{vk}(-c_e^{vk+t})E^{vk+t}\right)\right]\\
&\quad -(1-a)\sum_{t=s}^{k+s-1}\left[(d)^{t-s}\sum_{v=0}^{\infty}\left[(d)^{vk}(-c_e^{vk+t})\left[1-(1-a)^{v}\right]E^{vk+t}\right]\right]\\
&= (1-a)\sum_{t=s}^{\infty}\left((d)^{t-s}(-c_e^{t})E^{t}\right)-N\\
&= \sum_i LST^{i,P}-N,
\end{aligned}
$$

where $N = (1-a)\sum_{t=s}^{k+s-1}\left[(d)^{t-s}\sum_{v=0}^{\infty}\left[(d)^{vk}\left(-c_e^{vk+t}\right)\left[1-(1-a)^v\right]E^{vk+t}\right]\right]$.

Note that, with $a \in (0,1)$ the inner parenthesis $[1-(1-a)^v] < 1$ for $v > 0$. Therefore, since the two parts of the expression are otherwise equal (see third equation), the second part must be smaller than the first part. Thus, we must have $N < \sum_i LST^{i,P}$. $N > 0$ is obvious for $a < 1$.

B Appendix

The numerical model is an extention of the model used in Rosendahl (2008), where the number of firms is fixed.

The following cost function (in addition to the fixed cost F) is used for an endogenous number of identical firms:

$$c^{i,t}\left(q^{i,t}, e^{i,t}\right) = C_0\left(q^{i,t}\right)^2\left(C_1 + \left(\overline{r} - \frac{e^{i,t}}{q^{i,t}}\right)^2\right),$$

where $\overline{r}$ can be interpreted as the unconstrained emissions rate. Note that any deviation from this rate (positive or negative) is costly. Further, with no constraint on emissions, this is a simple quadratic cost function with marginal cost equal to $2C_0C_1q^{i,t}$. The model considers a closed product market with a linear inverse demand function: $p^t = A - B^0(1-\mu)^t Q^t$, where $Q^t = \sum_i q^{i,t}$ and μ is the annual growth rate of demand for a given price. Firms will enter (exit) the market if the present value of expected profits are positive (negative). The model parameters are presented in Table B1.

Table B1. Parameter values in the simulation model

Parameter	C_0	C_1	m^0	$\overline{r}$	A	B^0	μ	d	k	F
Value	88.9	0.56	100	1	500	4	0.02	0.91	5	10

The investment cost I is calibrated so that the initial number of firms is 100 (to be in line with Rosendahl (2008) - for calibration details, we refer to that paper). In the figures 1 and 2 we have assumed that the aggregate emissions target is held constant over time as long as the emission trading scheme lasts.

C Appendix

The numerical model is written in GAMS, and here we provide the GAMS code used:

```
$TITLE  Updated grandfathering with entry and exit
*       Date: 2008 --- K.E.Rosendahl and H. Storrøsten

set      t       number of periods       /0*200/;

alias (t,tt);

scalars
         C0      parameter cost function                  /88.868/,
         C1      parameter cost function                  /0.56263/,
         B       parameter demand function                /4/,
         A       parameter demand function                /500/,
         r       emission intensity BaU                   /1/,
         my      growth in demand function                /0.02/,
         delta   discount rate                            /0.1/,
         k       year between emission and allocation     /5/,
         m0      number of firms year 0                   /100/,
         E0      percentage emission constraint year 0    /0.99/,
         au      auction rate                             /0.5/,
         gamma   annual change in emission constraint     /1/,
         F       fixed annual cost                        /10/;

parameter ET(t) Emission target;

free variables
         obj             objective,
         op(t)           operating surplus in firm,
         opold(t)        operating surplus in old firm,
         opnew(t)        operating surplus in new firm,
         npv(t)          net present value of operating surplus from per t;

positive variables
         e(t)            emission in firm,
         q(t)            production in firm,
         c(t)            production costs in firm,
         p(t)            price,
         s(t)            quota price,
         LST             lump sum transfer to old firms,
         I               investment cost,
         m(t)            number of firms;

equations
         e_bau(t)        emission function bau,
         e_eq(t)         emission constraint,
         p_eq(t)         demand function,
         q_eq(t)         production function,
         c_eq(t)         cost function,
         sP_eq(t)        quota price function: pure grandfathering,
         sU1_eq(t)       quota price function: updating,
         sU2_eq(t)       quota price function: updating last periods,
         opnew_eq(t)     oper. surp. func. new firms: pure&upd. grandfathering,
         opoldP_eq(t)    oper. surplus func. old firms: pure grandfathering,
         opoldU1_eq(t)   oper. surplus func. old firms: updating,
         opoldU2_eq(t)   oper. surplus func. old firms: updating first k years,
         npvP_eq(t)      net present value function: pure grandfathering,
         npvU_eq(t)      net present value function: updating,
         LSTP_eq         lump sum transfer function: pure grandfathering,
         LSTU_eq         lump sum transfer function: updating,
         inv_bau(t)      investment condition in bau solution,
         inv_eq(t)       investment condition,
         inv2_eq(t)      investment condition no. 2,
         close_eq(t)     closedown condition,
         obj1_eq         objective function
         ;

* Sets BaU emissions to cost min. level
e_bau(t)..       e(t) =e= r*q(t) ;
```

```
* Total emissions target is allocated on all firms
e_eq(t)..         e(t) =e= ET(t)/m(t) ;

* Demand function: price is linear function of total production
p_eq(t)..         p(t) =e= A - B*((1-my)**(ord(t)-1))*(m(t)*q(t)) ;

* Price equals marg. prod. costs
q_eq(t)..         p(t) =e= 2*C0*( q(t)*C1 + r*(r*q(t) - e(t)) ) ;

* Production cost function
c_eq(t)..         c(t) =e= C0*q(t)*q(t) *
                  ( C1 + r**2 - 2*r*e(t)/q(t) +  (e(t)/q(t))**2 ) ;

* Quota price equals marginal abatement costs (pure grandfathering)
sP_eq(t)..         s(t) =e= -2*e(t)*C0 + (2*C0*A*r + 4*C0*C0*r*r*e(t)) /
                  (2*C0*C1 + 2*C0*r*r + m(t)*B*((1-my)**(ord(t)-1))) ;

* Relationship between quota price and MAC under updating
sU1_eq(t)$(ord(t) le (card(t)-k))..
                  s(t) - ((1+delta)**(-k))*s(t+k)*(gamma**k)*(1-au) =e=
                  -2*e(t)*C0 + (2*C0*A*r + 4*C0*C0*r*r*e(t)) /
                  (2*C0*C1 + 2*C0*r*r + m(t)*B*((1-my)**(ord(t)-1))) ;

* Quota price equals MAC last k periods under updating
sU2_eq(t)$(ord(t) gt (card(t)-k))..
                  s(t) =e= -2*e(t)*C0 + (2*C0*A*r + 4*C0*C0*r*r*e(t)) /
                  (2*C0*C1 + 2*C0*r*r + m(t)*B*((1-my)**(ord(t)-1))) ;

* Oper. surplus for new firms under pure and updated grandfathering (and BaU)
opnew_eq(t)..   opnew(t) =e= p(t)*q(t) - c(t) - e(t)*s(t) - F ;

* Operating surplus for old firms under pure grandfathering (and BaU)
opoldP_eq(t)..    opold(t) =e= p(t)*q(t) - c(t) -
                  (e(t)-e("0")*ET(t)/ET("0")*(1-au))*s(t) - F ;

* Operating surplus for old firms under updating except first k years
opoldU1_eq(t)$(ord(t) gt k)..    opold(t) =e= p(t)*q(t) - c(t) -
                  (e(t)-e(t-k)*(gamma**k)*(1-au))*s(t) - F ;

* Operating surplus for old firms under updating first k years
opoldU2_eq(t)$(ord(t) le k)..    opold(t) =e= p(t)*q(t) - c(t) -
                  (e(t)-e("0")*ET(t)/ET("0")*(1-au))*s(t) - F ;

* Net present value new firms under pure grandfathering (and BaU)
npvP_eq(t)..      npv(t) =e= sum(tt$(ord(tt) ge ord(t)),
                  (1+delta)**(ord(t)-ord(tt))*opnew(tt)) ;

* Net present value new firms under updating
npvU_eq(t)..      npv(t) =e=
                  sum(tt$((ord(tt) ge ord(t)) and (ord(tt) lt (ord(t)+k))),
                  (1+delta)**(ord(t)-ord(tt))*opnew(tt)) +
                  sum(tt$(ord(tt) ge (ord(t)+k)),
                  (1+delta)**(ord(t)-ord(tt))*opold(tt)) ;

* Net present value of lump sum transfer under pure grandfathering
LSTP_eq..         LST =e= sum(t,
                  ((1+delta)**(-ord(t)))*e("0")*ET(t)/ET("0")*(1-au)*s(t)) ;

* Net present value of lump sum transfer under updating
LSTU_eq..         LST =e= sum(t$(ord(t) le k),
                  ((1+delta)**(-ord(t)))*e("0")*ET(t)/ET("0")*(1-au)*s(t)) ;

* Investment condition in BaU-scenario
inv_bau(t)..       npv(t) =e= I ;

* Investment condition when emission constraint is set
* Either no investments (m unchanged) or investment costs equal npv
inv_eq(t)$(ord(t) lt card(t))..  (npv(t+1) - I)*(m(t+1)-m(t)) =e= 0 ;
```

```
* Make sure npv never exceeds investment costs
inv2_eq(t)..     npv(t) =l= I ;

* Make sure npv is never negative
close_eq(t)..    npv(t) =g= 0 ;

* Objective function - arbitrary objective variable
obj1_eq..        obj =e= -sum(t,npv(t)*(1+delta)**(-(ord(t)-1))) ;

model bau /q_eq, e_bau, p_eq, c_eq, npvP_eq, opnew_eq, inv_bau, obj1_eq/ ;

model mfixed /e_eq, p_eq, q_eq, c_eq, sP_eq, npvP_eq, opnew_eq,
                 opoldP_eq, obj1_eq/;

model pure /e_eq, p_eq, q_eq, c_eq, sP_eq, opnew_eq, opoldP_eq, npvP_eq,
                 LSTP_eq, inv_eq, inv2_eq, close_eq, obj1_eq/;

model updated /e_eq, p_eq, q_eq, c_eq, sU1_eq, sU2_eq, opnew_eq, opoldU1_eq,
                 LSTU_eq, opoldU2_eq, npvU_eq, inv_eq, inv2_eq, close_eq,
obj1_eq/;

q.lo(t) = 0.0001;
e.lo(t) = 0.0001;
p.lo(t) = 0.0001;
m.lo(t) = 1;
c.lo(t) = 0.0001;

* Solve BaU model with no quota price and m(0)=100 to find investment cost
* that is consistent with m(0)=100
s.fx(t) = 0;
m.fx("0") = 100;

* Set an arbitrary value to initialize ET(t) before BaU solution
ET(t)=100;

solve bau using nlp minimizing obj;

* Fix investment cost
I.fx = I.l;

* Set emissions constraint
ET(t) = m.l("0")*e.l("0")*E0*(gamma**(ord(t)-1)) ;

* Make the quota price free
s.lo(t) = 0.0001;
s.up(t) = inf;

* Run first pure grandfathering with m fixed over entire horizon to check
m.fx(t) = 100;

solve mfixed using nlp minimizing obj;

* Run pure grandfathering solution with m(0) fixed at 100,
* but endogenous afterwards
m.lo(t) = 1;
m.up(t) = inf;
m.fx("0") = 100;

solve pure using nlp minimizing obj;

* Run updated grandfathering solution
solve updated using nlp minimizing obj;
```